MASS-MOV: The 2.5D approach for Slow Moving Landslide

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Fundamental equations:

Mass and momentum balance

1. \[ \frac{\partial h}{\partial t} + \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) vh = 0 \]

2a. \[ \rho gh \left\{ \sin \alpha_x \cos \alpha_x + \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) v \frac{E}{\rho} - \left[ \cos^2 \alpha_x \tan \phi' \left( \frac{1}{\rho gh} \left( \tau_c + \eta \left( \frac{\partial v_x}{\partial z} \right) \right) \right) \right\} = 0 \]

2b. \[ \rho gh \left\{ \sin \alpha_y \cos \alpha_y + \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) v \frac{E}{\rho} - \left[ \cos^2 \alpha_y \tan \phi' \left( \frac{1}{\rho gh} \left( \tau_c + \eta \left( \frac{\partial v_y}{\partial z} \right) \right) \right) \right\} = 0 \]

Dependent on pore water pressure changes

Gravity force G (2D)  Lateral pressure P (2D)  Flow resistance (Coulomb viscous) (2D)
Model assumption

\[
\frac{\partial h}{\partial t} + \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) \mathbf{v}h = 0
\]

- One-phase homogeneous material with rheological properties
- Uncompressible solid with constant, homogeneous density \( \rightarrow h = f(\text{mass only}) \)
- No source or sink of material
Model assumption

Lateral pressure $P$ (m/s$^2$):

$$
\frac{\partial P}{\partial t} + \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) v P = -\frac{1}{\rho} \cdot E \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) v
$$

$$
\forall \ P_{\text{min}} < P < P_{\text{max}}
$$

$$
P_{\text{min}} = \frac{1 - \sin \varphi'}{1 + \sin \varphi'} \cdot \partial \left( \xi + h \right) \partial x
$$

$$
P_{\text{max}} = \frac{1 + \sin \varphi'}{1 - \sin \varphi'} \cdot \partial \left( \xi + h \right) \partial x
$$

- Acceleration due to internal pressure changes is due to strain, considering a certain elasticity of the landslide (although this certainly violates our assumption of an incompressible material)

- The value of $P$ ranges between a maximum and a minimum value, which depend on the Rankine state of the landslide
Model assumption

Velocity (m/s):

\[ v = \rho H d \frac{1}{\mu} \left( G - P + S \right) \]

where: \( d = h - h_{crit} \)

and \( h_{crit} = \frac{\tau_c}{\rho (Q + P - S_f - Sc)} \)

Driving and resistance acceleration (m/s²):

\[ G = g \left( \sin \alpha \cos \alpha \right) \]

\[ S = \frac{\tau_0}{\rho h} + g \cos^2 \alpha \tan \phi' \]

\[ \phi' = \tan^{-1} \left( \frac{1 - \frac{u}{\rho h}}{\tan \phi} \right) \]

- no inertia: if the driving forces stop, the landslide would stop; we don't conserve the velocity; instead we calculate a new velocity at each time step
- the resisting term S, opposing movement, includes yield strength due to cohesion and a frictional term
- the apparent friction angle depends on the pore pressure ratio; hence, the changes in pore pressure control to a great extent the movement of the landslide
Model assumption

Pore water pressure (Pa):

\[
\begin{cases}
\frac{\partial u_{ex}}{\partial t} + \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) v u_{ex} = \zeta + A E \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) v \\
\frac{\partial u_{ba}}{\partial t} + \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) v u_{ba} = 0
\end{cases}
\]

\[u_{tot} = u_{ba} + u_{ex} F_{diss}\]

• No loss nor gain of water during the simulation
  → The initial pore water pressure \(u_{ba}\) stays constant and travels with the mass.
  → excess in pore pressure are only due to strain (differential velocity), considering a certain elasticity of the landslide. The excess of pore pressure is then progressively dissipated

• Gain of excess pore pressure occurs only in compression mode

• Strain causes a redistribution of water within the pores of the landslide, only in the vertical direction; there is no lateral migration of water from a cell to another cell
Model assumption

Dissipation of pore water pressure $F_{diss}$:

\[ u_{ex} = u_{ex}^{tc} \left( u_{ex}^{t-1} e^{T_v} \right) \]

\[ T_v = \frac{4 h^2_s}{\pi^2 c_v} \]

where:

- $u_{ex}$ = excess pore pressure at consolidation time $t_c$ (Pa)
- $T_v$ = time factor (s)
- $h_s$ = height of saturated layer (m)
- $c_v$ = consolidation coefficient ($m^2/s$)

- We assume that excess pore pressure dissipates following an exponential law based on Therzaghi’s one dimensional consolidation theory assuming parallel flow along the slope (Hutchinson, 1986).
Numerical resolution

Similar to the 1D approach, but with some differences...

Landslide thickness \( h \): 

\[
dh = \text{div}(vh) \text{ if } h = 0 \text{ and } dh > 0
\]

\[
dh = 0 \text{ if } h \neq 0 \text{ or } dh < 0
\]

\[
h = h - dTdh
\]

Avoid taking material from outside the landslide

Forces (resistance term \( S \)): 

\[
D_y = G_y + P_y
\]

\[
D_x = G_x + P_x
\]

\[
S = Sc + Sf
\]

1. \( S \) is calculated as a scalar

\[
S_y = -D_y \frac{S}{D}
\]

2. \( S \) is decomposed in two components strictly opposite to the driving forces \( D \)

\[
S_x = -D_x \frac{S}{D}
\]
Numerical resolution

Dissipation of pore water pressure $F_{\text{diss}}$:

Depending on the height of the saturated layer $h_s$, the pore water pressure can dissipate during several time steps.

![Diagram showing dissipation of pore water pressure over time steps](image_url)

- Pore water pressure is still dissipating after one time step.
- All dissipated after one time step.
Numerical resolution

Dissipation of pore water pressure $F_{\text{diss}}$:

- $t_c$ is the cumulated consolidation time ($= n \times \text{timestep } \Delta t$, $n$ is an integer).
- $t_c$ is conservative (considers the previous timesteps and travels with the mass).

When the cell is in compression mode, $t_c$ becomes equal to the timestep $\Delta t$.

\[
\begin{aligned}
  t_c &= t_c + \Delta t (\text{div}(t_c \, \mathbf{v}) + 1) & : \text{for extension mode (div >0)} \\
  t_c &= \Delta t & : \text{for compression mode (div <0)} \\

t_c &\text{ is 0 at the first time step}
\end{aligned}
\]
Numerical resolution

Dissipation of pore water pressure $F_{\text{diss}}$:

$t_{\text{ini}} = \Delta t$

\[
\begin{align*}
    t_c &= \Delta t; \text{ if } \text{div } \mathbf{v} < 0 \\
    t_c &= t_c; \text{ if } \text{div } \mathbf{v} > 0
\end{align*}
\]

\[
\begin{align*}
    F_{\text{diss}} &= 0; \text{ if } Tv = 0 \\
    F_{\text{diss}} &= \exp\left(\frac{-t_c}{Tv}\right); \text{ if } Tv \neq 0
\end{align*}
\]

\[
\begin{align*}
    du_{ex} &= \text{div}(vu) + (1 + A)\text{div}(v)E; \text{ if } \text{div}(v) < 0 \\
    du_{ex} &= \text{div}(vu); \text{ if } \text{div}(v) \geq 0
\end{align*}
\]

\[
\begin{align*}
    u_{ex} &= u_{ex} - dTdu_{ex}; \\
    u_{ba} &= u_{ba} - dT\text{div}(vu_{ba}); \\
    u_{tot} &= u_{ba} + u_{ex} F_{\text{diss}}; \\
    t_c &= t_c + dT \text{div} (\mathbf{v} t_c) + 1
\end{align*}
\]
Sensitivity analysis

Time step:

\[ \frac{20}{T_d} \rightarrow n \text{ where } n \in \mathbb{N} \]

20 days simulations with different Time steps
Sensitivity analysis

![Graph showing sensitivity analysis of various parameters.](image-url)

- Red line: Viscosity
- Blue line: Slope
- Black line: Rho
- Magenta line: Cohesion

The graph illustrates the cumulated displacement changes (%) against parameter changes (%).
Sensitivity analysis

E modulus $5 \times 10^6$ kPa  In compression mode very high excess of PWP are generated when realistic value of E modulus are used.

- Compression mode
  - Generation of excess PWP
  - Dissipation of excess PWP

- Extension mode
  - Decrease of the effective friction angle

Initial pore water pressure
Excess of pore water pressure due to strain
Sensitivity analysis

E modulus $5 \times 10^4$ kPa
Simulation of a synthetic case

# Mud rheology
- Density of the solid mass = 2000 kg.m$^{-3}$
- Density of the water = 1000 kg.m$^{-3}$
- Yield stress = 1000 Pa
- Viscosity = 3.10$^{10}$ Pa.s
- Angle of internal friction = 25°
- A = 0.5
- E = 5.10$^{9}$ Pa
- Cv = 0.0002 m$^2$.s$^{-1}$

# Timestep control
- Timeslice = 8640 s (1/10 of a day)
- Timesteps = 1000

# Other constants
- Gravity = -9.8 m.s$^{-2}$
- Cell size = 5 m
Simulation of a synthetic case
Simulation of a synthetic case

Lateral pressure (ms²)

PWP (Pa)
Simulation of a synthetic case

Mass balance

\[ h_{\text{mean ini}} = 5.28 \text{ m} \]

\[ h_{\text{mean final}} = 5.32 \text{ m} \]

\[ \left( \frac{h_{\text{mean final}} - h_{\text{mean ini}}}{h_{\text{mean ini}}} \right) = +0.87\% \]

Still a mass balance problem
Simulation of a synthetic case

Vertical velocity profile

\[ n = \text{number of nodes} \]

\[ \Delta h = \frac{h}{n} \quad i \in [0, n] \]

\[ v_{xi} = (n - i) \Delta h^2 \frac{1}{\eta} \left( \Phi_x - Sc - Sf'_x \right) \]

\[ v_{yi} = (n - i) \Delta h^2 \frac{1}{\eta} \left( \Phi_y - Sc - Sf'_y \right) \]

\[ \text{disp}_i = \text{disp}_{i-1} + dT \sqrt{v_{xi}^2 + v_{yi}^2} \]
Simulation of a synthetic case

Vertical velocity profile

Cumulated displacement (m) vs. n
Simulation of a synthetic case

Vertical velocity profile
Simulation of a synthetic case

Vertical velocity profile

\[ h_{\text{crit}} = \frac{\tau_c}{\rho \mathcal{D} + P - Sf - Sc} \]

\[ \tau_c = 1 \text{ kPa} \]

\[ \tau_c = 10 \text{ kPa} \]

\[ \tau_c = 50 \text{ kPa} \]

\[ \tau_c = 100 \text{ kPa} \]
Conclusions (1/3)

- The elastic behavior of the landslide is predominant.
- Strain rate together with E modulus generate huge amount of $U_{ex}$ realistic?

Consequence:

- A more visco-plastic behavior is necessary.
- Maybe avoid the E modulus? (contradicts the first assumption of the model $\rightarrow$ uncompressible...)
Conclusions (2/3)

- Need an increase in pore pressure by loading

- Need to include a B parameter for skempton law. (here, a B = 1 is only for saturated conditions…)

\[ \Delta p = B_d \sigma + A_d \Delta \tau \]
\[ \Delta p = B_d \rho g \Delta h (\cos^2 \alpha + A_d \sin \alpha \cos \alpha) \]

…
Conclusions (3/3)

Application on a real case → Super Sauze mudslide: calibration and validation